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Inferential Control of Processes:

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Part I. Steady State Analysis and Design

Methods are presented for the design of a static estimator which infers unmeasurable product qualities from secondary measurements. The secondary measurements are selected so as to minimize the number of such measurements required to obtain an accurate estimate which is insensitive to modeling errors. Unlike previous work, the number of secondary measurements can be fewer than the number of unmeasured disturbances. If the statistics of the disturbances and/or measurement noise are available, this information can be incorporated into the design procedure to obtain an optimal static estimator.

The design procedure is illustrated by application to a simulated industrial debutanizer. Data for the simulation were supplied by the Marathon Oil Company. Deviations in bottoms product quality are compared for the current control policy (maintenance of a stage temperature at its set point) and the inferential control system with the column subjected to representative feed composition disturbances. Results show that inferential control based on four, five, or six tray temperature measurements improves the steady state control performance by as much as 400%.

SCOPE

An inferential control system uses measurements of secondary process outputs, such as temperatures, to infer the effect of unmeasurable disturbances on primary process outputs, such as product quality. The control system uses its inference to adjust the control effort to counteract the effect of the unmeasurable disturbances on the product quality. Inferential control can be viewed as an extension of feedforward control which infers the effect of measurable disturbances on the product quality and adjusts the control effort to counteract the effect of the measured disturbances. Feedforward and inferential control can be applied simultaneously.

These three papers present a strategy for the design of linear inferential control systems for processes which operate about a sequence of steady states. The unmeasurable disturbances are assumed to drift from one mean level to another at a rate such that the process will operate about one or another steady state condition most of the time. The above assumption is, of course, motivated by our picture of the operation of most chemical and petroleum process.

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Because the process operates mainly in the steady state, the most critical part of the design of an inferential control system is the design of the steady state estimator. Part I of these papers is devoted to the design of the steady state estimator. Particular attention is paid to the selection of secondary measurements which minimize estimator sensitivity to modeling errors and measurement noise. Part I concludes by describing an application of the design procedures to design a static inferential control system to control product composition of a simulated industrial debutanizer.

The dynamic structure of linear inferential control systems is discussed in Part II. We show that the response and stability characteristics of such systems are similar to those of feedforward control systems. Dynamic compensation of the steady state estimator is achieved with simple lead-lag networks, which are designed in an ad hoc fashion. Part II concludes by using an inferential control system to control both the overhead and bottoms product of a simulated, dynamic, multicomponent distillation column.

Part III presents rigorous methods for the design of suboptimal dynamic estimators and provides theoretical justification for splitting up the design of the estimator into static and dynamic phases. The design of optimal linear estimators via the methods of Kalman and Wiener are shown to yield complex estimators which will generally be less attractive to implement than the simpler suboptimal estimators. Optimal and suboptimal estimators are applied to the same simulated distillation column used in Part II.

CONCLUSIONS AND SIGNIFICANCE

Methods are given for the design of linear inferential control systems which are stable, easy to implement, and relatively insensitive to modeling errors. Application to control the product compositions of simulated multicomponent distillation columns shows that the inferential control systems so designed substantially improve product quality control over that achieved by temperature feedback control systems.

The results achieved with inferential control applied to simulations give promise that such systems will be effective in controlling the product quality of many processes when the product quality cannot be economically measured at a rate suitable for use in feedback control systems. The excellent stability properties of multivariable inferential control systems should make them attractive alternatives to multivariable feedback control of two or more process outputs, even when the product quality can be measured.

Many process control problems share the following three characteristics: the process is subject to unmeasurable disturbances that vary slowly or change abruptly but infrequently, the primary output to be controlled is not easily measured, and there are other process outputs that are relatively easily measured. A typical example is the control of multicomponent distillation process. Here, the major input disturbances are variations in the feed composition, temperature, and flow rate. The quantity to be controlled is the bottoms and/or top product composition. Gas chromatographs can be used to monitor the product composition but are expensive, difficult to maintain, and introduce undesirable time delays in the feedback control loop (Scott, 1968; Villalobos, 1968). Industry practice is to maintain a constant temperature on one of the trays close to the product withdrawal location using feedback control. This is not always satisfactory, since maintaining a constant temperature does not necessarily maintain a constant composition.

Inferential control uses selected temperature and flow measurements to estimate and control the effect of unmeasurable input disturbances on the product composition. Let us consider the nature of input disturbances in a little more detail in order to motivate a model for these disturbances. Input disturbances frequently arise from changes in the operation of units upstream from the process. Because of the disturbances, the process moves from one steady state to another. For small input disturbances, the process can be assumed to behave linearly. The unmeasured output y and the measurements θ can be related to the input disturbances u as follows:

$$\theta = A^T u \tag{1}$$

$$y = b^T u \tag{2}$$

We may treat θ and y as perturbation variables (changes from the desired operating point). A and b are constant matrices.

When the number of disturbances u is less than the number of measurements θ, one can solve for u from Equation (1) (or its subset) and then compute y exactly from Equation (2). This approach was taken by Weber and Brosilow (1972) in their study of the use of multiple temperature measurements for distillation column control. Easterday (1973) extended Weber's work to include methods for dealing with cases where the number of measurements were less than the number of input disturbances. The next section reviews Easterday's work as well as some more recent developments (Joseph, 1975).

STEADY STATE ESTIMATION AND CONTROL

Let the disturbances u be modeled as zero-mean random variables with covariance ϕ_{uu}. Without loss of generality we can assume $\phi_{uu} = I$, since otherwise new variables u' can be defined such that $u' = Cu$ and $\phi_{u'u'} = I$. The steady state estimation problem is to obtain an estimate $\hat{y}$ of the process output from the secondary measurements θ. Since u is a random variable, θ and y are also random variables, and the problem reduces to the estimation of one random variable in terms of another. Using a least-square criterion, the problem is to minimize the expectation of the square error [that is, $(y - \hat{y})^2$]. If the input disturbances are Gaussian, the least-square estimate of y is given by (Rhodes, 1971)

$$\hat{y} = (\phi_{y\theta}\, \phi_{\theta\theta}^{-1})\, \theta \tag{3}$$

where $\phi_{\theta\theta} = E\{\theta\theta^T\} = A^T A$. $E\{\cdot\} \equiv$ the expected value

of $\{\cdot\}$:

$$\phi_{y\theta} = E\{y\theta^T\} = b^T A$$

Let

$$\alpha \equiv [A^T A]^{-1} A^T b \tag{4}$$

then the least-square estimate of y is given by

$$\hat{y} = \alpha^T \theta \tag{5}$$

The constant vector α will be referred to as the estimator. It can also be obtained from the least-square solution of the set of equations

$$A\alpha \cong b \tag{6}$$

The fact that best least-squares estimate of y is a linear combination of θ is a direct consequence of the fact that the inputs u are Gaussian distributed. However, even if u is not gaussian, α is still the best linear estimator (Rhodes, 1971).

An underlying assumption in the above result is that the matrix inverse in Equation (4) exists. If it does not, then some of the measurements are linearly dependent [rank (A) $< k$, where k is the number of measurements]. Linear dependence in the measurements can be avoided by dropping the dependent measurements.

The estimation error $y - \hat{y}$ is given by

$$\begin{aligned} e &\equiv y - \hat{y} \\ &= (b^T - \alpha^T A^T)u \\ &= r^T u \quad \text{where } r \equiv b - A\alpha \end{aligned} \tag{7}$$

The expected values of the error and error squared are

$$E\{e\} = 0 \tag{8}$$

$$E\{e^2\} = r^T r \tag{9}$$

Equation (8) states that α is an unbiased estimator, and Equation (9) states that the expected mean-square error $E\{e^2\}$ is zero if and only if $b = A\alpha$; that is, b belongs to the space spanned by the columns in A.

Associated with each estimator we define a quantity called the projection error, which is given by

$$\text{projection error} \equiv (||r||/||b||) \tag{10}$$

The projection error ranges from 0 to 1 and is a measure of how accurate the estimate will be.

Sensitivity of the Estimate to Input Statistics

One of the parameters required in the construction of the estimator is the covariance of the input disturbance vector. Since accurate measurements of these statistics are rather difficult, the designer will often be forced to estimate the input covariance matrix. The question naturally arises as to how errors in input statistics will affect the estimate. The error in the estimate of y is given by Equation (7). We no longer assume that $\phi_{uu} = I$, and therefore the expected value of the square error in estimation is

$$E\{e^2\} = r^T \phi_{uu} r$$

If the largest element in ϕ_{uu} is denoted by M, then

$$E\{e^2\} \leq M \left\{ \sum_{i=1}^{n} |r_i| \right\}^2$$

The quantity M is the square of the largest expected deviation in the disturbances u. If $||r||$ is small, $\sum_{i=1}^{n} |r_i|$ will be small, and hence $E\{e^2\}$ will be small. The conclusion is that the more accurate the estimator is, the less sensitive it is to changes in input statistics. In the case that $r = 0$, the estimator does not depend on the input statistics at all.

Effect of Measurement Noise on the Estimator

Let the measurement θ be corrupted with random measurement noise η where $E\{\eta\} = 0$. Then

$$\begin{aligned} \theta &= A^T u + \eta \\ \phi_{\theta\theta} &= A^T A + \phi_{\eta\eta} + A^T \phi_{u\eta} + \phi_{\eta u} A \\ \phi_{y\theta} &= b^T A + b^T \phi_{u\eta} \end{aligned} \tag{11}$$

and from Equation (3)

$$\hat{y} = [b^T A + b^T \phi_{u\eta}]\,[A^T A + \phi_{\eta\eta} + A^T \phi_{u\eta} + \phi_{\eta u} A]^{-1}\,\theta$$

The expected error and error squared are

$$E\{e\} = 0 \tag{12}$$

$$\begin{aligned} E\{e^2\} = b^T b - 2b^T(A + \phi_{u\eta})\,\alpha + \alpha^T(A^T A \\ + \phi_{\eta u} A + A^T \phi_{u\eta} + \phi_{\eta\eta})\alpha \end{aligned} \tag{13}$$

If the input disturbances are uncorrelated with the measurement noise, then

$$\phi_{\eta u} = \phi_{u\eta} = 0$$

and the presence of noise on the measurements increases only the quadratic term in (13) for all values of the estimator α. Thus, the effect of measurement noise is to increase the uncertainty in the estimate of the process output $\hat{y}$.

Effect of Measured Variables

Let m denote the changes in measured variables that affect the process. The model can now be written as

$$\theta = A^T u + Pm \tag{14}$$

$$y = b^T u + cm \tag{15}$$

Define new variables

$$\begin{aligned} \theta^* &= \theta - Pm = A^T u \\ y^* &= y - cm = b^T u \end{aligned} \tag{16}$$

The equations derived in the previous section can be applied to the estimation of y^* from θ^*, and the solution is

$$\hat{y} = \alpha^T[\theta - Pm] + cm$$

with α the same as before.

Control Using Secondary Measurements

The control problem is to manipulate the control effort in such a way as to minimize the expected value of the square of the deviation from the set point, that is, $E\{y^2\}$. On the surface, this problem appears to be different from that of choosing an estimator α to minimize the expected value of the square of the estimation error, that is, $E\{e^2\}$. However, as we show below, the two problems are the same.

Let m denote the single manipulated variable used to control the scalar output y. We seek a control law of the form

$$m = \mu^T \theta \tag{17}$$

such that $E\{y^2\}$ is minimized. Substituting for m in Equations (14) and (15), we get

$$\begin{aligned} y &= b^T u + c\mu^T \theta \\ &= b^T u + c\mu^T[I - P\mu^T]^{-1} A^T u \end{aligned} \tag{18}$$

Let a quantity α be defined as

$$\alpha^T \equiv -c\mu^T[I - P\mu^T]^{-1}$$

Then (18) can be written as

$$y = (b - A\alpha)^T u \tag{19}$$

The value of α which minimizes $E\{y^2\}$ is, of course

$$\alpha = (A^TA)^{-1}A^Tb$$

which is the same as given by (4) for the best estimate of y. The parameter μ is obtained from the estimator α as

$$\mu^T = (-c + \alpha^TP)^{-1}\alpha^T \tag{20}$$

and m is given by

$$\begin{aligned} m &= \mu\theta \\ &= (-c + \alpha^TP)^{-1}\alpha^T\theta \end{aligned} \tag{21}$$

A difficulty with computing the control effort from Equation (21) is that the indicated inverse need not exist. It is, therefore, better to rearrange (21) into the following form:

$$m = -c^{-1}\alpha^T[\theta - Pm] \tag{22}$$

The term $\theta - Pm$ is independent of m when θ is given by (14). In general, for nonlinear systems, the term $\theta - Pm$ will be independent of m only over a small range, after which it will be weakly dependent on the control effort. Thus, for nonlinear systems, a reasonable iterative scheme for obtaining the control effort is given by

$$m_{k+1} = c^{-1}\alpha^T(\theta(m_k) - Pm_k) \tag{23}$$

Nonlinear Systems

Chemical process systems are generally nonlinear. In this section, some methods of incorporating nonlinearities into the estimator construction are discussed. Consider the following nonlinear process

$$\theta = f(u)$$
$$y = g(u)$$

where f and g are nonlinear (vector) functions. Expanding around the origin [$f(0) = 0$ and $g(0) = 0$ since θ and y are perturbation variables], we get

$$\theta = f_u u + u^T f_{uu} u + \text{higher order terms}$$

$$y = g_u u + u^T g_{uu} u + \text{higher order terms}$$

where $f_u \equiv \partial/\partial u\ f$, $f_{uu} \equiv \partial^2/\partial u^2\ f$. The model used in earlier sections is obtained by neglecting second- and higher-order terms. Dropping third- and higher-order terms, we obtain a quadratic system of equations. Define a new vector $v = (u_1, u_2 \ldots u_n, u_1^2, \ldots u_1u_2, u_2^2 \ldots u_n^2)$. The system can now be represented as

$$\theta = A^Tv \tag{24}$$

$$y = b^Tv \tag{25}$$

with A and b suitably defined The techniques of earlier sections can be applied to Equations (24) and (25). The penalty for this trick is that the dimension of the input vector for the new system is considerably larger than for the linear system.

Another way of incorporating nonlinearities is to define new variables as nonlinear functions of the original process variables in such a way that the process is linear in the new variables. For example, when we deal with small compositions (for example, $y = 10^{-3}$), it often pays to define a new variable which is the logarithm of the composition (for example, $z = \log y$) and, in this way, avoid the possibility of predicting negative compositions (see Joseph et al., 1976).

Distributed Parameter Systems

The steady state linear input-output model for systems distributed in a single space dimension can be written as

$$\theta(x) = \sum_{i=1}^{n} a_i(x)u_i(x) \tag{26}$$

$$y = b^Tu \tag{27}$$

where $0 \leq x \leq 1$.

It is possible to discretize the system in the space dimension x and then apply the techniques of the previous sections. However, some additional insight into the estimation problem is obtained by considering the problem of exactly estimating y by sampling $\theta(x)$.

Let $\phi_1(x), \phi_2(x), \ldots, \phi_m(x)$ represent a basis for the space spanned by the vectors $a_1(x), a_2(x), \ldots, a_n(x)$. Here, $m \leq n$. Then, the functions $a_j(x)$, $j = 1, 2, \ldots, n$ can be written as

$$a_j(x) = \sum_{i=1}^{m} d_{ij}\phi_i(x) \quad j = 1, 2, \ldots, n$$

The measurement $\theta(x)$ becomes

$$\theta(x) = \sum_{i=1}^{m} (d_i^Tu)\phi_i(x)$$

where

$$d_i^T = (d_{i1}, d_{i2}, \ldots, d_{in})$$

Then we have the following theorem.

Theorem. The output $y = b^Tu$ can be estimated exactly if and only if b belongs to the space spanned by the vectors $d_1, d_2, \ldots, d_m$. The vectors $d_1, d_2, \ldots, d_m$ are linearly independent.

Proof is given in Joseph (1975). This theorem implies that the space spanned by the vectors $d_1, d_2, \ldots, d_m$ is independent of the basis chosen. The quantities d_1^Tu can be obtained by sampling $\theta(x)$ at m locations (Murray-Lasso, 1966).

Let d_i^Tu be denoted by $\hat{\theta}_i$. The problem of estimating y from $\theta(x)$ is equivalent to the problem of estimating b^Tu from θ_i, $i = k, 2, \ldots, m$. By comparing this to the lumped parameter estimation problem, we have

$$\hat{y} = b^TD(D^TD)^{-1}\hat{\theta}$$

where $D = [d_1, d_2, \ldots, d_m]$. In practice, it may be difficult to determine the exact number of independent vectors in $a_i(x)$, $i = 1, 2, \ldots, n$ because of modeling errors. Even when the number of independent vectors are known, it may be difficult to obtain as many measurements. In this case, the measurements may be selected using the procedure given later.

Sensitivity of the Estimator to Modeling Errors

Consider a change α_ϵ caused in the estimator α due to modeling errors A_ϵ and b_ϵ in A and b, respectively. The estimator equation for the case without measurement noise can be written as

$$(A + A_\epsilon)(\alpha + \alpha_\epsilon) = b + b_\epsilon$$

It can be shown that (Weber and Brosilow, 1972)

$$\frac{||\alpha_\epsilon||}{||\alpha||} \leq \text{cond}\ (A)\ \frac{||b_\epsilon - A_\epsilon\alpha||}{||b||} \tag{28}$$

where

$||\alpha||$ = a norm of the vector α

cond $(A) \equiv$ the condition number of the matrix A

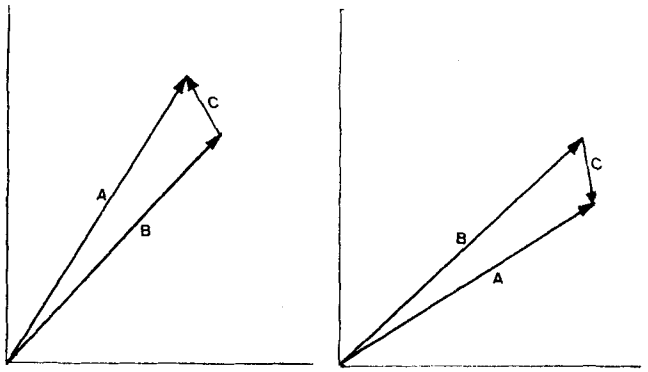

Fig. 1

$$\equiv ||A|| \cdot ||A^{-1}||$$

$$= \sqrt{\frac{\text{maximum eigenvalue of } A^TA}{\text{minimum eigenvalue of } A^TA}}$$

if the norm is taken to be the Euclidian norm; that is $||\alpha|| = \left(\sum_i \alpha_i^2\right)^{1/2}$

Equation (28) gives the lowest possible bound on the propagation of modeling errors (A_ϵ and b_ϵ) into an error α_ϵ in the estimator. Experience indicates that the condition number should be less than 100 to avoid unacceptable sensitivity to modeling errors. The lowest possible condition number is unity.

The condition number is also a measure of the sensitivity of the estimate to modeling errors as shown in Equation (29) (Joseph, 1975):

$$E\{y_\epsilon^2\} \leq [\text{cond } (A)]^2 \, ||b_\epsilon - A_\epsilon \alpha||^2 \qquad (29)$$

Two conditions which yield models having high condition numbers are disturbances affecting several secondary measurements in almost the same way, and disturbances affecting some measurement much more strongly than others. In the first case, the columns of the matrix A [see Equation (1)] are almost, but not quite, linearly dependent. The estimator tries to extract needed information from the difference between two or more of these almost dependent vectors, and, of course, the information obtained is then very sensitive to changes in the vectors. For example, the vector A in Figure 1 is almost parallel to the vector B. Small changes in A change the sign and magnitude of the difference vector C.

In the second case, the estimator tries to extract information from measurements which are only weakly influenced by the disturbances. This situation again places heavy dependence on the accuracy of the model, and this dependence is reflected by a high condition number.

Since any linear model is valid only over a limited operating range, one cannot place stringent accuracy conditions on the process model. Thus, it is necessary to select secondary measurements which lead to a model which has a moderate or low condition number. The next section presents several algorithms for selecting measurements which make the estimator α relatively insensitive to modeling errors.

SELECTION OF SECONDARY MEASUREMENTS

Two conditions necessary for good estimator performance are low relative error and low model condition number. The relative error is defined as the square root of the ratio of the expected squared error in the estimate divided by the expected squared value of the output y; that is

$$\text{relative error} \equiv \left\{\frac{E\{e^2\}}{E\{y^2\}}\right\}^{1/2} \qquad (30)$$

The relative error defined above is a measure of the accuracy of the estimator, assuming a perfect model. Defining the disturbances, as before, so that the covariance matrix $\phi_{uu} = I$, the expected squared value of the output is $||b||^2$ (that is, $E\{y^2\} = ||b||^2$). The expected squared value of the estimation error $E\{e^2\}$ is given by Equation (13) when the measurements are noisy and by Equation (9) when the measurement noise is negligible. In the latter case, the relative error is the same as the projection error defined in Equation (10).

Ideally, one seeks the smallest set of measurements which have both a low relative error and a low sensitivity to modeling errors, as reflected by a low model condition number. However, the relative error is generally a decreasing (strictly nonincreasing) function of the number of measurements, while the condition number is generally an increasing function of the number of measurements. Thus, there is frequently a trade off between estimator accuracy as measured by relative error and estimator sensitivity as measured by condition number. Of course, the objective is to design an estimator which performs well on the actual process.

The condition number associated with a single measurement is always unity. Thus, if one can find a single measurement which has a low relative error, the measurement selection problem is solved. If no such single measurement exists, then one must look for multiple measurements. Before embarking on such a search, however, one should check the relative error using all possible measurements. If this relative error is unacceptably large, then it will not be possible to construct an acceptable estimator for the product quality without enlarging the measurement set.

If the total number of secondary measurements are small, then one can enumerate all possible measurement combinations, calculate the relative error and condition number, and select the set or sets having acceptably low relative errors and condition numbers. As a rough rule, an acceptable measurement set is one for which the relative error is less than 0.05 and condition number is less than 100.

When the number of available secondary measurements is large, it will be difficult to select measurements by enumerating the various sets and evaluating each of them. For example, consider a typical distillation process with forty-five plates. There are forty-five possible single temperature measurement locations, 990 two temperature measurement sets, and 14 190 three temperature measurement sets. Each relative error calculation involves the solution of a set of equations, and each condition number calculation requires the evaluation of the eigenvalues of the matrix A^TA. It is thus impractical to apply the enumeration technique to this problem.

An alternative to enumeration is to add measurements one at a time. This method will generally not yield the optimum answer, but one is not seeking the optimum measurement set but rather simply a satisfactory set. The measurements are selected as follows. First, select the single measurement which yields the smallest relative error. If this relative error is not satisfactory, another measurement from the remaining set is selected such that the addition of this new measurement reduces the relative error the most. If there are K measurements, this step requires the calculation of $K - 1$ relative errors. The condition number of the resulting two measurement

TABLE 1

	No. of measurements	Measurement locations	Projection error × 100	Condition number
Maximum condition number = 1 000	1	14	12.2	1
	2	14, 1	8.9	3.0
	3	14, 1, 24	7.6	3.6
	4	14, 1, 24, 28	5.2	27.2
	5	14, 1, 24, 28, 2	3.9	73.3
	6	14, 1, 24, 28, 2, 13	3.6	952.4
	7	14, 1, 24, 28, 2, 13, 16	3.3	959.0
Maximum condition number = 100	6	14, 1, 24, 28, 2, 35	3.7	95.0
Maximum condition number = 50	5	14, 1, 24, 28, 35	4.2	30.2

set is then calculated. If it is below the maximum allowable condition number set by the designer, then the relative error may be further reduced by adding another measurement. If, on the other hand, the condition number exceeds the maximum, the measurement selected most recently is replaced by the measurement which increases the relative error least. The procedure is repeated until a satisfactory condition number is obtained or all measurements are exhausted. In the latter case, the relative error can be reduced only by relaxing the constraint on the condition number.

Example

The above procedure was applied to select measurements on a debutanizer column with thirty-five stages and twenty input disturbances. (The operation of the process itself is discussed in detail in a related article by Joseph et al., 1976). The results are summarized in Table 1. The selection procedure was applied with three different constraints on the condition number. In each case, the procedure was terminated when no further measurements could be added to the set without violating the constraint on the condition number. The first four measurement locations are the same in each set because the condition number associated with these locations is smaller than the smallest constraint (that is, cond. No. ≤ 50).

A few trends are evident from Table 1. The condition number increases monotonically with the number of measurements selected. The decrease in projection error with the addition of new measurements becomes less and less as the number of measurements increase, which is an indication that there is less and less new information regarding the input disturbances in the additional measurements. Also, notice that measurements which are spatially near each other, as with the sets containing four and five measurements, do not necessarily have a high model condition number.

Based on the condition numbers and projection errors given in Table 1, a reasonable choice of measurements would be that given by the set with three measurements.

Measurement Selection in Distributed Parameter Systems

The measurement selection procedure suggested below is the distributed analog of the procedure given above for lumped systems. The model for the distribution system is given by Equations (26) and (27):

$$\theta(x) = \sum_{i=1}^{n} a_i(x)\, u_i(x) \tag{26}$$

$$y = b^T u \tag{27}$$

A single measurement location is obtained by scanning the functions $a_i(x)$ over the range of x so that the vector A_1, given by

$$A_1^T(x_1) \equiv [a_1(x_1), a_2(x_1) \ldots a_n(x_1)]$$

yields the smallest relative error. In the case of no measurement noise, the quantity $||b - A_1(x_1)\alpha||$ is a minimum over all values of x and α. Additional measurements are chosen such that the relative error is minimized each time a measurement is added. Each stage of the algorithm requires an optimization over the scalar variable x. As before, measurement locations which increase the condition number of the set of vectors (A_1, A_2 . . .) beyond a preselected value are not included in the measurement set. The procedure stops when no measurement location can be found which does not increase the condition number beyond the preselected limit.

APPLICATION TO MULTICOMPONENT DISTILLATION

The principles of steady state inferential control set forth in the previous section were applied to the control of a debutanizer column operated by the Marathon Oil Company The study was conducted using a digital simulation of the tower and plant operating data collected over a period of about 1 yr. Details of the study are given by Joseph (1975) and Joseph et al. (1976). The main results of the study are summarized in this section.

The Control Problem

The control objective is to maintain a constant mole fraction of isobutane in the bottoms product in spite of feed composition variations. Control is achieved by manipulating the bottoms flow rate. The feed to the column has twenty components ranging from hydrogen to a pseudo component which boils at 686°K (775°F). Thirty feed stock compositions selected from plant operating data over 1 yr were used to represent typical disturbances in feed composition. The average of these thirty feeds was taken as the nominal feed composition around which the column operates. The temperature measurements were limited to the six measurements already available on the column.

A linear input-output model of the process was constructed by perturbing the twenty disturbances (variation in the feed flow rate of each component) one at a time. The manipulated variable was taken as the ratio of the bottoms flow rate to the feed flow rate.

TABLE 2. CONTROL STUDY

Steady state deviation of isobutane as a percent of the target value (target value of isobutane in bottoms = 7×10^{-4} mole fraction)

Feed sample No.	Constant T_4 control	Estimator 1 (6 temp.)	Estimator 2 (best 5 temp.)	Estimator 3 (best 4 temp.)
1	−244.81	−54.61	−50.06	−40.02
2	−72.34	−46.85	−42.86	−41.83
3	−11.40	−5.14	−1.50	0.08
4	−133.59	−16.49	−25.62	−33.90
5	−144.60	1.59	−10.80	−47.93
6	1.64	35.25	34.82	33.36
7	−21.43	44.08	43.80	43.51
8	−37.42	−72.48	−42.71	−2.37
9	−228.86	20.53	12.71	−0.73
10	−288.49	−32.90	−56.82	−101.82
11	−141.38	7.71	6.98	8.41
12	−26.95	−20.00	13.31	1.14
13	28.49	15.99	16.62	27.99
14	47.11	61.66	−30.04	−33.17
15	11.23	40.62	36.42	24.27
16	28.67	42.41	41.15	36.23
17	33.06	38.59	36.05	23.83
18	48.39	18.00	10.07	−9.63
19	−425.15	6.03	−12.01	−37.37
20	60.00	−39.78	−53.38	−77.00
21	88.00	19.96	9.05	−16.89
22	23.74	22.98	16.62	1.61
23	−290.30	−20.07	15.00	−25.00
24	71.00	21.31	14.76	0.44
25	−93.17	29.62	29.83	29.48
26	84.00	−16.09	−9.23	22.18
27	57.69	12.83	14.29	13.27
28	−138.68	30.85	24.10	9.61
29	82.43	−50.00	66.00	−66.00
30	−53.94	3.63	−5.77	−24.92
Mean avg.	−55.56	3.30	3.36	−9.44
Std. dev.	142.44	33.91	31.75	36.93
Cond. no.	—	275.40	46.80	7.67
Proj. error %	—	5.10	5.11	5.57

Perturbation tests showed that the fraction of isobutane, in the bottoms product, is a highly nonlinear function of the input disturbances. This nonlinearity is probably due to the fact that the mole fraction of isobutane in the bottoms is very small. The target value is a mole fraction of only 7×10^{-4}. A robust linear model was obtained by defining a new product composition as the logarithm of the isobutane mole fraction. The linear model then becomes

$$\theta = A^T u + Pm$$

$$z = b^T u + cm$$

where

$$z = \log y$$

Three estimators were constructed using all six, the best five, and the best four temperature measurements, respectively. Here best refers to the temperature set that gives the smallest projection error. Each estimator is constructed from a linear combination of the selected temperature measurements and the ratio of the bottoms to feed flow rate. The bottoms to feed flow rate is adjusted to drive the estimated mole fraction of isobutane in the bottoms product to the target value.

The above estimator contains no dynamic compensation because the objective of this first paper is only to study the steady state behavior of inferential control systems. Parts II and III of this sequence of papers show how to add dynamic compensation to the static estimator. As can be seen from the examples presented in Part II, implementation costs of an inferential control system using even six temperature measurements are quite modest.

Results of the Control Study

The column was subject to all thirty different feed conditions (each corresponding to a set of input disturbances) and the steady state error in control was calculated as

$$\% \text{ error in control} = \frac{(y - y_d)}{y_d} \times 100$$

where y_d is the desired or target value of the mole fraction of isobutane in the bottoms and y is the actual value obtained. The inferential control system is compared to the existing stage 4 temperature feedback control system. The results are shown in Table 2. The standard deviation of the percent error calculated over the thirty feeds gives an indication of the efficiency of various control schemes. It is seen that all three inferential controllers do a better job of keeping the bottoms isobutane constant than the temperature feedback control. These estimators also exhibit less bias from the target composition.

The study also shows the significance of low model condition number. Estimator 1, using all six measurements, has a large condition number (275), indicating that there is a strong coupling between the measurements. Hence, it can be expected to be very sensitive to modeling errors. This is indeed the case, since estimator 2, using only five measurements, does better. Note that estimator 2 has a smaller condition number (46.8) but about the same projection error. Hence, adding a measurement to estimator 2 deteriorates its performance. On the other hand, removing a measurement from estimator 2 we get estimator 3, which is less sensitive to modeling errors (condition number of only 7.6) but which has a 10% larger projection error. The increase in projection error has wiped out the decrease in the percent error due to the decreased sensitivity of the estimator. Thus, the condition number and the projection error together must be used to guide in selecting the number of measurements and their locations.

Other studies (Joseph et al., 1976) show that the inferential control system allowed one to move accurately from a target value of 7×10^{-4} to a target value of 6×10^{-3}. Further, the inferential system performs better in the presence of variations in feed temperature and flow and reflux flow than does the single temperature feedback system.

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NOTATION

A, b, P, c = process matrices
$E\{\}$ = expected value of $\{\}$
k = iteration number
m = vector of measured process inputs
u = vector of unmeasured process disturbances
y = vector of unmeasured process outputs or product qualities
y_d = vector of target values for y
y_ϵ = error in y induced by modeling errors

Greek Letters

α = estimator matrix
η = vector of random measurement noise

μ = vector of control law coefficients
θ = vector of unmeasured process outputs
ϕ_{uu} = covariance matrix of the disturbances u
$\phi_{\theta\theta}$ = covariance matrix of the measurements θ
$\phi_{y\theta}$ = $E\{y\theta^T\}$, the cross correlation matrix of y with θ
$||[\cdot]||$ = norm of $[\cdot]$

Superscripts

$\hat{}$ = estimated value
T = transpose

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Part II. The Structure and Dynamics of Inferential Control Systems

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The static estimator of part I is incorporated into a dynamic control system. The system structure minimizes output feedback and thereby achieves stability even in the event of significant modeling errors. This inherent stability permits the use of simple and economical lead-lag compensation for the estimator and controller portions of the system. The overall system response to disturbances is similar to that of feed-forward control systems.

The proposed control system structure is used to infer and control the overhead and bottoms product compositions of a simulated multicomponent distillation. Product compositions are inferred from selected stage temperature and process flow measurements. Inferential control system response to various disturbances is comparable or superior to that of a tuned composition feedback control system for both single product control and for simultaneus overhead and bottoms product control.

It is generally necessary to add some form of dynamic compensation to the static inferential control system obtained by the methods of Part I in order to obtain an operable control system. What is meant by an operable control system will vary from process to process, but commonly it will mean that the process lines out after a disturbance in less than three or four time constants without large excursions from the set point. Also implied in the idea of operability is the requirement that the control system be robust. That is, its response must not be seriously degraded by shifts in process operating conditions.

The block diagram structure for inferential control systems proposed in this work attempts to achieve an operable control system by assuring control system stability. The most critical transfer functions can be obtained from readily available plant data and/or simple plant tests. Owing to the inherent stability of the proposed control systems, the dynamic compensation of the estimator and controller need not be perfect, and simple lead-lag networks will usually suffice.

Inferential control systems can also be used in combination with feedforward and feedback control systems. Typical block diagrams for such configurations are presented later.

The dynamic behavior of inferential control systems with various types of modeling errors is discussed later. The important result in this section is that stability can almost always be assured by making sure that modeling errors are always in the appropriate direction.

We will present the results of applying inferential control, with ad hoc dynamic compensation to control the overhead and/or bottoms product compensation of a simulated multicomponent distillation column. Compositions are inferred from selected stage temperatures and process flows.

THE BLOCK DIAGRAM

A typical process block diagram is given to the right of the dotted line in Figure 1. All process inputs and outputs are vector quantities. Generally, the dimension of the product quality $y(s)$ and the control effort $m(s)$ vectors will be small, and frequently these variables are scalars. The number of secondary measurements $\theta(s)$ is under the control of the designer (see Part I), but economic considerations will usually limit the number of measurements to the order of 10 or less. The number of unmeasured disturbances $u(s)$ which enter a process varies widely with the process and can range from several hundred (for exam-

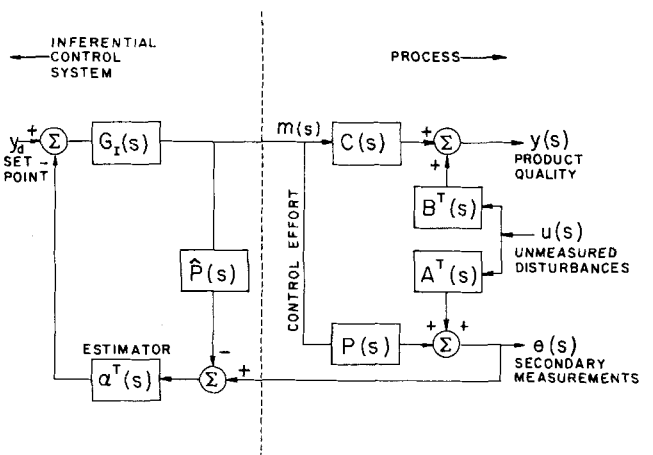

Fig. 1. Inferential control system.